

11-4 Irrational Square Roots

Objective: To simplify radicals and to find decimal approximations of irrational square roots.

Vocabulary

Irrational numbers Real numbers that can't be expressed in the form $\frac{a}{b}$, where a and b are integers. Their exact values can't be expressed as either terminating or repeating decimals.

Property

Property of Completeness Every decimal represents a real number, and every real number can be represented by a decimal.

Example 1 Simplify: a. $\sqrt{256}$ b. $\sqrt{50}$ c. $2\sqrt{80}$ d. $\sqrt{704}$

Solution a. $\sqrt{256} = \sqrt{4 \cdot 64}$ Factor within the radical sign.
 $= \sqrt{4} \cdot \sqrt{64}$ Use the product property of square roots.
 $= 2 \cdot 8$ Simplify.
 $= 16$

$$\begin{aligned} \text{b. } \sqrt{50} &= \sqrt{25 \cdot 2} \\ &= \sqrt{25} \cdot \sqrt{2} \\ &= 5\sqrt{2} \end{aligned}$$

$$\begin{aligned} \text{c. } 2\sqrt{80} &= 2\sqrt{16 \cdot 5} \\ &= 2 \cdot 4\sqrt{5} \\ &= 8\sqrt{5} \end{aligned}$$

$$\begin{aligned} \text{d. } \sqrt{704} &= \sqrt{64 \cdot 11} \\ &= 8\sqrt{11} \end{aligned}$$

Simplify.

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|-------------------|------------------|-------------------|-------------------|--------------------|
| 1. $\sqrt{27}$ | 2. $\sqrt{20}$ | 3. $\sqrt{72}$ | 4. $\sqrt{32}$ | 5. $\sqrt{48}$ |
| 6. $\sqrt{45}$ | 7. $\sqrt{196}$ | 8. $\sqrt{80}$ | 9. $2\sqrt{63}$ | 10. $4\sqrt{98}$ |
| 11. $7\sqrt{28}$ | 12. $4\sqrt{40}$ | 13. $\sqrt{441}$ | 14. $\sqrt{289}$ | 15. $3\sqrt{50}$ |
| 16. $12\sqrt{50}$ | 17. $\sqrt{729}$ | 18. $\sqrt{432}$ | 19. $8\sqrt{75}$ | 20. $2\sqrt{90}$ |
| 21. $\sqrt{147}$ | 22. $\sqrt{288}$ | 23. $\sqrt{4225}$ | 24. $5\sqrt{800}$ | 25. $5\sqrt{1025}$ |

11-4 Irrational Square Roots (continued)

Example 2 Approximate $\sqrt{396}$ to the nearest hundredth. Use your calculator or the table at the back of your textbook.

Solution

$$\begin{aligned}\sqrt{396} &= \sqrt{2^2 \cdot 3^2 \cdot 11} \\ &= \sqrt{2^2} \cdot \sqrt{3^2} \cdot \sqrt{11} \\ &= 6\sqrt{11}\end{aligned}$$

From the table: $\sqrt{11} \approx 3.317$
 $6\sqrt{11} \approx 6(3.317) \approx 19.902$
 Therefore $\sqrt{396} \approx 19.90$.

Example 3 Approximate $\sqrt{0.6}$ to the nearest hundredth. Use your calculator or the table at the back of your textbook.

Solution

$$\sqrt{0.6} = \frac{\sqrt{60}}{\sqrt{100}} = \frac{\sqrt{60}}{10} \approx \frac{7.746}{10} = 0.7746$$

Therefore $\sqrt{0.6} \approx 0.77$.

In Exercises 26–37, use your calculator or the table at the back of the book. Approximate each square root to the nearest tenth.

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|--------------------|--------------------|----------------------|----------------------|
| 26. $\sqrt{600}$ | 27. $\sqrt{200}$ | 28. $-\sqrt{800}$ | 29. $-\sqrt{500}$ |
| 30. $-\sqrt{2700}$ | 31. $-\sqrt{2200}$ | 32. $\pm\sqrt{6600}$ | 33. $\pm\sqrt{4800}$ |

Approximate each square root to the nearest hundredth.

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|-----------------|-----------------|-------------------|-------------------|
| 34. $\sqrt{56}$ | 35. $\sqrt{32}$ | 36. $-\sqrt{0.7}$ | 37. $-\sqrt{0.2}$ |
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Mixed Review Exercises

Find the indicated square roots.

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|-----------------------------|-------------------|--|
| 1. $\sqrt{100}$ | 2. $-\sqrt{144}$ | 3. $\sqrt{\frac{9}{25}}$ |
| 4. $-\sqrt{\frac{36}{121}}$ | 5. $\sqrt{154^2}$ | 6. $\sqrt{\left(\frac{2}{5}\right)^2}$ |

Simplify.

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|---------------------|--------------------|----------------------------------|
| 7. $(13x)^2$ | 8. $(2y^3z^6)^2$ | 9. $(x + 2y)^2$ |
| 10. $[10(a + 1)]^2$ | 11. $(9a^3b^7c)^2$ | 12. $(4z^2 + 3y^3)(4z^2 - 3y^3)$ |